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ANALOG-TO-DIGITAL CONVERTER DIFFERENTIAL NONLINEARITY ERROR CORRECTION IN BUILDING CONDUCTANCE HISTOGRAMS

In nanowire electrical conductance quantization studies, conductance histograms are built from a large number of conductance curves of conductance stepwise variations in time. The conductance curves are obtained by means of a digital oscilloscope; as each digital oscilloscope has an analog-to-digital converter (ADC) whose differential nonlinearity (DNL) error strongly affects the obtained conductance histograms. The effect of the DNL error can be corrected through the procedure described in this paper. Moreover, measurement results of the DNL error for three digital oscilloscope models and the long-term DNL error variations for one oscilloscope model are presented

Keywords: differential nonlinearity, analog digital conversion, electrical conductance, quantized conductance, nanowires

1. INTRODUCTION

Since 1988, when electrical conductance quantization was measured in a two-dimensional electron gas quantum point contact (QPC) [1, 2], electric charge transport in solids has been the object of revived scientific interest. In 1993, electrical conductance quantization was found to occur in gold nanowires fabricated with STM at room temperature [3] and low temperature [4]. Further studies showed nanowires could form also in a less-sophisticated setup: between two vibrating wires (1995) [5] or between relay contacts (1997) [6]. However, fabrication of stable nanowires of width comparable with Fermi wavelength is a very difficult task [7].

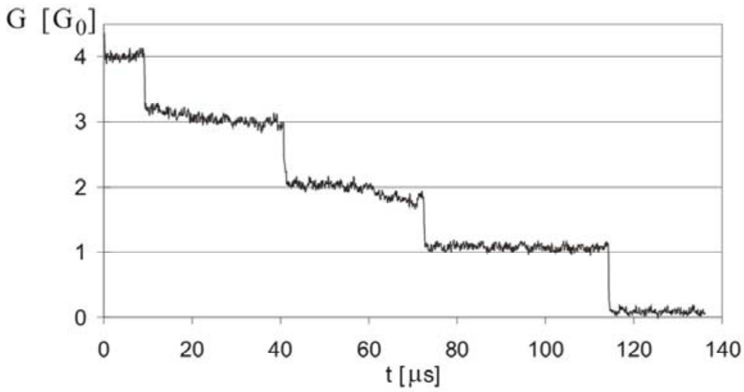


Fig. 1. Electrical conductance in an Au nanowire during elongation, this process exhibits conductance quantization steps.

The method most commonly used in nanowire studies is based on the effect of nanowire formation occurring between two macroscopic electrodes during their separation [8, 9]. The electrodes are first brought to collide; when one of them is subsequently retracted at an appropriately set speed, nanowires form between the separating electrodes. As a result of electrode retraction, the nanowires are stretched until they break, and in the end, only one nanowire remains unbroken. Measured just before the breakage, and plotted versus time, its electrical conductance will show steps (as depicted in Fig. 1), which are the evidence of nanowire electrical conductance quantization. When the nanowire is being stretched, its electrical conductance takes quantized values $K \cdot G_0$ ... $4 \cdot G_0$, $3 \cdot G_0$, $2 \cdot G_0$, $1 \cdot G_0$; G_0 can be found from the following formula:

$$G_0 = \frac{2e^2}{h} = \frac{2}{R_K}, \tag{1}$$

e denoting the elementary charge, h being Planck's constant, and R_K standing for von Klitzing's constant.

Figure 1 corresponds to the ideal case of ballistic (scatter-free) electron transport in a nanowire. If electron scattering occurs in the nanowire, extra electrical conductance plateaus will appear between $k \cdot G_0$ values. Therefore, in nanowire electrical conductance quantization studies, conductance histograms [10, 6] are built from a large number of conductance curves of conductance stepwise variations in time. The positions of peaks (local maximums) in the obtained histograms provide information on properties of the quantization process under investigation. Histogram-based studies of electrical conductance quantization have been performed in gold [5, 6, 10, 11], copper [6, 12], platinum [13], palladium [14], magnetic metal [15] and alloy [16, 17] nanowires. The conductance curves are obtained by means of a digital oscilloscope; as each digital oscilloscope has an analog-to-digital converter (ADC) whose differential nonlinearity (DNL) error strongly affects the obtained conductance histograms; correction of this error is necessary for the histograms to be reliable. If neglected, the DNL error can produce false peaks in the resulting incorrect histograms.

2. ELECTRICAL CONDUCTANCE MEASUREMENTS AND HISTOGRAM BUILDING

Figure 2 shows a schematic diagram of the experimental setup used for electrical conductance measurements in conductance quantization studies. Nanowires are formed between electrodes A and B of the studied metal. Measured during electrode separation - *i.e.* during nanowire stretching - the electrical conductance G between the electrodes corresponds to the nanowire electrical conductance. The digital storage oscilloscope (DSO), used for signal sampling and recording, must be triggered at a right moment for the measurements to cover the process of stretching and breaking of the last remaining nanowire. The arbitrary waveform generator is used to control the movement of the A-electrode. The digital storage oscilloscope and the arbitrary waveform generator are controlled by a PC through an IEEE-488 interface.

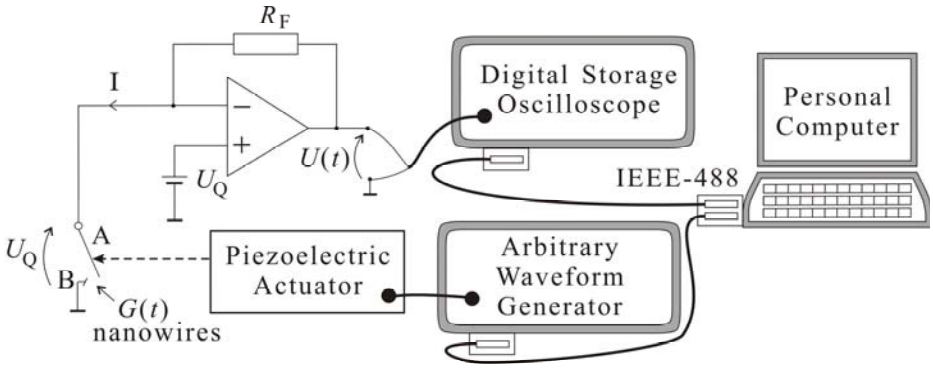


Fig. 2. Schematic diagram of the experimental setup used to study nanowire formation between macroscopic metallic electrodes

With an appropriate measurement software, the required number of conductance curves can be obtained to provide the basis for building conductance histograms. The nanowire current I is converted into voltage by an operational amplifier in the current amplifier circuit. The voltage on the operational amplifier's inverting input, equal to that applied to its noninverting input (U_Q), produces nanowire polarization, and consequently current I. With resistor R_F adjusted so that U_Q is negligible with respect to U_F, the following relation is satisfied:

$$U = R_F I = R_F U_Q G. \tag{2}$$

The electrical conductance (in G_0 units) between the electrodes can be found from the following formula:

$$G(t) = \frac{U(t)}{R_F U_Q G_0}, \tag{3}$$

where R_F is the resistance introduced in the operational amplifier feedback, U_Q is the nanowire bias voltage and $U(t)$ denotes the time-variable voltage on the current amplifier output during electrode separation (nanowire stretching). This provides the $U(t)$ signal to be processed by the oscilloscope's ADC, which is done through two operations: signal sampling and quantization. In this case, sampling consists in converting the time-continuous signal $G(t)$ into a time-discrete signal G_n which can be expressed as follows:

$$G_n = \frac{U(nT_S)}{R_F U_Q G_0}, \tag{4}$$

where n = 1, 2, 3, ..., N; N is the number of samples taken, and T_S is the sampling period. As a result of the subsequent signal quantization, the voltage values are represented by a finite set of values resulting from the number of ADC's quantization levels (codes):

$$U \in \{ U_0, U_1, U_2, \dots, U_{W-1} \}, \tag{5}$$

where W is the number of ADC's codes. In digital oscilloscopes, the following relation between U and the raw result of analog-to-digital conversion is satisfied:

$$U_w = \left(\left(w - Y_{ref}\right)Y_{inc}\right) + Y_{org}, \tag{6}$$

where w = 0, 1, 2, ...,W-1; Y_{ref}, Y_{inc} [V] and Y_{org} [V] are scaling coefficients specified by the oscilloscope manufacturer. The electrical conductance sample values that can be measured and used for plotting conductance curves are elements of the following set:

$$G \in \{\, G_0, G_1, G_2, \dots, G_{W-1} \}. \tag{7}$$

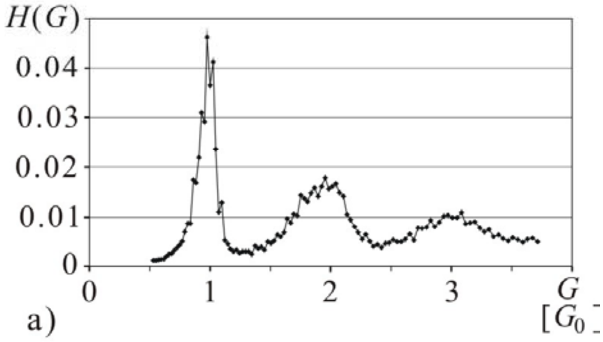
Conductance values G_w (w = 0, 1, 2, ... ,W-1), expressed in G_0 units, can be calculated from the following relation:

$$G_w = \frac{U_w}{R_F U_Q G_0}. \tag{8}$$

In order to build a histogram, P conductance curves are recorded, comprising N samples each. Therefore, as the result of signal G sampling and quantization, $P \times N$ samples are obtained in the measurement process, the sample values being elements of set (7). In the next step, samples recorded for each w are counted, and the resulting counts are used in building the histogram, whose points are defined by the following formula:

$$H(G_w) = \frac{\sum G_w}{NP}, \tag{9}$$

ΣG_w denoting conductance G_w sample count (the number of samples corresponding to conductance value G_w).



a)

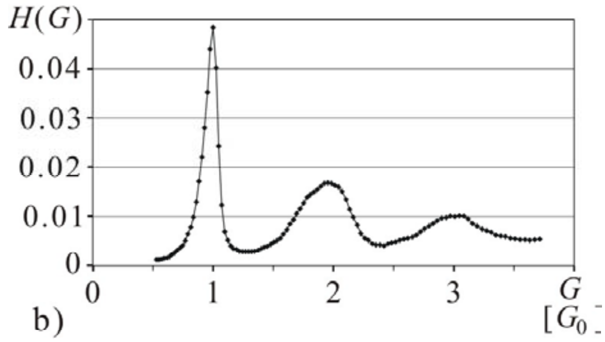


Fig. 3. Conductance histograms for gold nanowires built from 6000 curves without DNL error correction (a) and with DNL error correction.

Figure 3a shows a histogram of nanowire conductance G in the range from $0.5G_0$ to $3.75G_0$, built from 6000 conductance curves without DNL error correction. The effect of the DNL error can be corrected through the procedure described below.

3. DNL ERROR CORRECTION

In the ideal ADC transfer function, each code has a uniform width. Differential nonlinearity (DNL) specifies the deviation of any code in the transfer function from an ideal code width of 1 LSB. When an ADC is used for electrical conductance measurements, its DNL error affects the histograms built from the measurement results.

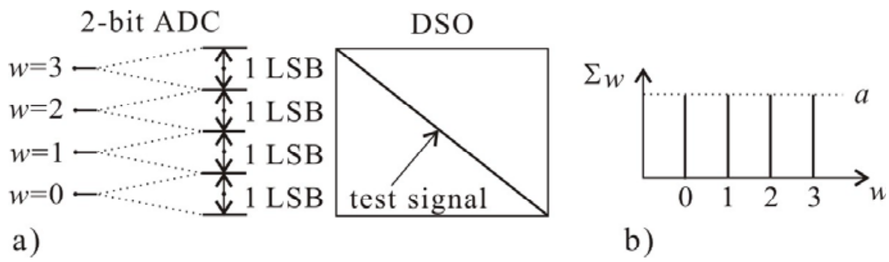


Fig. 4. Schematic representation of a DSO with a DNL error-free ADC (a) and a histogram obtained from test curves (b).

Figure 4a and 5a shows two oscilloscopes, each of them having a 2-bit ADC. The ADC in the oscilloscope shown in Figure 4a is DNL-error free, which means its code widths are uniform: 1 LSB for each code w. The ADC in the oscilloscope depicted in Figure 5a does have a DNL error; codes $w = 1$ and $w = 2$ have different code widths, equal to 5/4 LSB and 3/4 LSB, respectively. A test signal (a negative voltage ramp signal that sweeps the full-scale range of the oscilloscope's ADC) is applied to both oscilloscopes to get P test curves, obtained with N samples each. The samples corresponding to each ADC code are counted, the obtained count being denoted as Σw for each code w. Figures 4b and 5b show the resulting Σw histograms obtained by means of the oscilloscopes depicted in Figs. 4a and 5a. Sample counts Σw obtained by means of the oscilloscope with a DNL-error free ADC (Fig. 5a) will be equal and independent of w:

$$\Sigma_w = \frac{PN}{W} = a\,, \tag{10}$$

a denoting each of the equal sample counts obtained with the DNL-error free ADC; W is the number of ADC codes, P is the number of test signal measurements, and N is the number of samples per measurement. Sample counts *?w* obtained with the other oscilloscope, using an ADC with a non-zero DNL error (Fig. 5b), will depend on the corresponding code width: Σ = a for code $w = 0$ and $w = 3$, but for $w = 1$ and $w = 2$ the count will be different and equal to $\Sigma1 = (5/4)a$ and $\Sigma2 = (3/4)a$ samples respectively. In this ADC with a non-zero DNL error, the relative code widths can be found from the following relation:

$$r_w = \frac{a}{\Sigma_w}\,, \tag{11}$$

where a, the theoretical sample count in a DNL-error free ADC, can be found from (10); w is the code number, and Σw is the actual sample count obtained for code w from P test signal measurements with N samples taken per measurement.

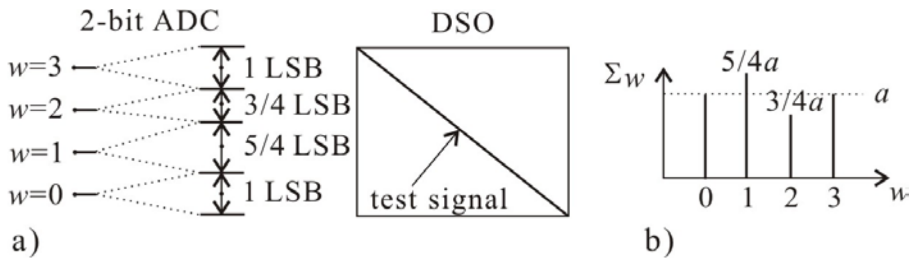


Fig. 5. Schematic representation of a DSO with a non-zero DNL error (a) and a histogram obtained from test curves (b).

Expressed in LSB units, the DNL error corresponding to each code w can be found from the following formula:

$$DNL_w = \frac{\Sigma_w - a}{a}\,. \tag{12}$$

In order to eliminate the effect of the ADC's DNL error on the conductance histogram defined in (9), each histogram value $H(Gw)$ should be multiplied by the corresponding relative code width rw, which can be found from (11). The following corrected formula for conductance histogram points is obtained through applying this correction procedure:

$$H(G_w) = \frac{\sum G_w}{NP} r_w. \tag{13}$$

Figure 3b shows a conductance histogram built on the basis of (13); it is to be compared with the histogram shown in Figure 3a, built from the same measurement results, but without DNL error correction. Constructed on the basis of (13), which involves DNL error correction, the histogram is smoother and constitutes a reliable basis for interpretation of measurement results.

4. DNL ERROR MEASUREMENTS

Table 1. Maximum values, minimum values and sums of the the DNL error for three DSO models.

DSO model	DNL_{max} [LSB]	DNL_{min} [LSB]	$\sum_{w=0}^{W} DNL_w$ [LSB]
LeCroy 9310CM	+0.28	+0.00037	16.31
HP 54602B	+0.51	-0.00077	18.27
HP 54600B	-0,61	+0.0011	18.99

DNL error measurements were performed for ADCs in three digital oscilloscope models: LeCroy 9310CM, Hewlett Packard 54602B and Hewlett Packard 54600B, with P = 6000 consecutive test signal curves generated for each of the studied ADCs, the number of samples per curve being N = 2500. The theoretical sample count, a, in a DNL-error free ADC with uniform code widths was calculated, and the actual sample counts Σw were found for each output code w = 0, 1, ..., 255 in the studied 8-bit ADCs. The DNL error was found on the basis of (12). The calculation results are shown in Table 1. The maximum DNL error values are found to be +0.28 LSB, +0.51 LSB and -0.61 LSB, in LeCroy 9310CM, HP-54602B and HP-54600B models, respectively. Long-term DNL error variations were examined as well. The test signal measurements were repeated at 6 months' intervals for the ADC in the LeCroy 9310CM oscilloscope model. The measurement and calculation results are shown in Table 2; for clarity reasons, only part of the ADC's full-scale range, from w = 215 to w = 220, is covered. The value and sign of the long-term DNL variations are found to vary with the output code w and to be random in nature.

Table 2. Long-term DNL error variations.

	time [month]					
	0	6	12	18	24	30
DNL for w = 215 [LSB]	-0.103	-0.0986	-0.1019	-0.10303	-0.104	-0.0999
DNL for w = 216 [LSB]	0.0463	0.0481	0.0472	0.04871	0.0496	0.0393
DNL for w = 217 [LSB]	0.00479	0.00469	-0.00319	-0.00023	0.00377	-8.33E-05
DNL for w = 218 [LSB]	0.0812	0.0817	0.0757	0.0821	0.0846	0.0755
DNL for w = 219 [LSB]	-0.0383	-0.0305	-0.0465	-0.0392	-0.0421	-0.0436
DNL for w = 220 [LSB]	0.0791	0.0829	0.0741	0.0707	0.0674	0.0731

5. CONCLUSIONS

The results obtained in this study prove that conductance histogram correctness is affected by the ADC's DNL error; however, the effect of this error can be eliminated through the above-described correction procedure. As conductance histograms are built on the basis of measurements performed by means of digital oscilloscopes, the relative code widths corresponding to each output code of the oscilloscope's ADC should be found, and the DNL error correction procedure applied for the histograms has to be reliable. Note that multichannel oscilloscopes use more than one ADC, which implies the necessity of performing relative code width measurements for each ADC. Differential nonlinearity is a specific parameter of ADCs; the DNL error is unique to each ADC specimen and its value and sign are random in nature. If the DNL error is above 1 LSB or below -1 LSB, code holes appear (*i.e.* some codes are missing) in the ADC's transfer function. Measurements performed by means of an ADC with code holes do not provide a reliable basis for histogram construction. Therefore, an ADC (an oscilloscope with ADC) should be examined for code holes before being used for such measurements. Moreover, our results prove that the DNL error is time-variable and its variations are random in nature. Therefore, the relative code width measurements should be repeated at intervals.

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KOREKCJA WPŁYWU BŁĘDU NIELINIOWOŚCI RÓŻNICZKOWEJ PRZETWORNIKA A/C W POMIARACH HISTOGRAMÓW PRZEWODNOŚCI

Streszczenie

W badaniach kwantowania przewodności elektrycznej w nanodrutach na podstawie dużej liczby zarejestrowanych skokowych zmian przewodności w funkcji czasu (przebiegów czasowych) wyznaczane są histogramy przewodności. Do pomiarów przebiegów czasowych używane są oscyloskopy cyfrowe. Parametrem wpływającym na poprawność wyznaczenia histogramu przewodności jest błąd nieliniowości różniczkowej przetwornika a/c znajdującego się w oscyloskopie. Wpływ tego błędu może być wyeliminowany poprzez zastosowanie procedury korekcyjnej opisanej w artykule. W artykule przedstawiono także wyniki pomiarów błędu nieliniowości różniczkowej dla trzech modeli oscyloskopów cyfrowych oraz długoterminowych zmian błędu nieliniowości różniczkowej dla wybranego oscyloskopu.